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Structure of quark diagrams*

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We give a prescription for calculating all quark diagrams associated with any given particle diagram. The amplitude of a multi-Regge quark diagram is shown to be equal to that term of the full amplitude which has discontinuities in the planar variables. Although this result holds only when exchange degeneracy is assumed, our prescription can also be used in the absence of exchange degeneracy. Some peculiarities related to the charge-conjugation structure of quark diagrams are discussed.

INTRODUCTION

All diagrams one uses for describing hadronic reactions are of two types: particle diagrams and quark diagrams. A particle diagram is any diagram whose lines represent objects with the quantum numbers of particles (such as external or virtual particles in Feynman diagrams, or exchanged Reggeons in Reggeon diagrams). A quark diagram is a diagram whose lines represent quarks.[1]

In this paper we discuss the following question: Given any particle diagram, what information can we obtain concerning all quark diagrams associated with it? Our conclusion is that once the expression for the particle diagram is given, it is possible to calculate in a straightforward way all the corresponding quark diagrams.

This conclusion is independent of the details of the rules for calculating the particle diagrams. The only essential assumption we need is that any particle (or Reggeon) line can be represented by a quark-antiquark pair. (We do not consider baryons in this paper.)

Our motivations for studying the quark topology of particle diagrams are the following:

a. Recently there has been much interest in the program of building the Pomeron and Reggeon contributions to the $2 \rightarrow 2$ amplitude, using production amplitudes $(2 \rightarrow n)$ as an input to the unitarity equation.[2,7,9,12] In this program the quark topology plays an important role in identifying the Reggeon and Pomeron terms in the resulting $2 \rightarrow 2$ amplitude.

b. Quark topology is important in studying the analytic structure of amplitudes. Diagrams with different quark topology are expected in some cases to represent terms with different analytic structure. For example, we do not expect any u-channel discontinuity in the s-t diagram of Fig. 1(a), or s-channel discontinuity in the u-t diagram [Fig. 1(b)]. (These expectations are motivated by the planar level of the dual model.)

c. Recently, a topological expansion (the $1/N$ expansion) has been suggested[3] for hadronic amplitudes. The order of each diagram in this expansion is determined by its quark topology.

In Sec. I we describe a general method for calculating all quark diagrams associated with any given particle diagram. For this purpose, we decompose each vertex into two components and find a relation between the two. We study the peculiar charge-conjugation properties of quark diagrams and their relation to the analytic structure. In Sec. II we discuss the relation of our method to the usual description of quark topology in terms of twisted propagators. In Sec. III we apply this method to the multi-Regge amplitude. We find the relation between the signature structure of the amplitude and its quark topology. A recursion formula is derived for any diagram with arbitrary quark topology. A prescription for calculating quark diagrams is given for the case where exchange degeneracy is broken.

The method described in this paper can be used[4] to find the contribution of interference terms to the unitarity relation of the above-mentioned program of studying the output Reggeon and Pomeron amplitudes. This may give us some insight into the $1/N$ expansion, since including all interference terms means solving the problem to all orders in $1/N$. We can then compare the noninterference solution, which is essentially the first order in $1/N$, with the full solution, in order to see to what extent the $1/N$ expansion is justified and how it depends on the value of N.

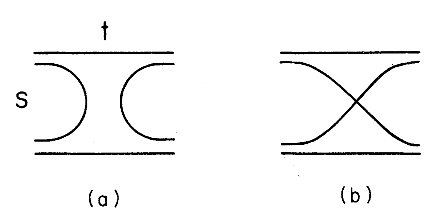

FIG. 1. (a) s-t quark diagram. (b) u-t quark diagram.

I. TWO-COMPONENT VERTEX

It is important to consider both particle and quark diagrams, because each type of diagrams by itself lacks some pieces of information. When we consider particle diagrams we lose track of the quark structure which is of great importance for the purposes just mentioned.

On the other hand, when we consider quark diagrams we do not have any information concerning quantum numbers such as angular momentum, parity, and charge conjugation of the particles which are produced or exchanged. Therefore, we first consider a given particle diagram where all internal and external particles (or Reggeons) are specified (so all the quantum numbers are well defined). Only then do we draw all quark diagrams associated with this specific particle diagram. The method of determining the expression for each quark diagram is independent of the rules for calculating the particle diagrams. In particular, this method is independent of the question over which set of particle diagrams we have to sum in order to get the physical amplitude.

We restrict the discussion to particle diagrams without baryons and with three-point vertices only. Consider any given particle diagram such as the one in Fig. 2(a). Our most important assumption is that any particle (or Reggeon) can be represented as a quark-anitquark pair, as is done in Fig. 2(b). This assumption implies that we do not allow Pomeron exchanges in Reggeon diagrams. Now we have to specify how to connect the quark lines at each vertex. But before doing so, a few remarks concerning the SU(N) structure of the problem are appropriate.

For simplicity we assume that SU(N) is a good symmetry. In the Appendix we show that our results are independent of this assumption. This assumption enables us to consider a world with only one type of quark ($N=1$). Once we know how to calculate quark diagrams for $N=1$, we can easily do it for any value of N. We can change the type of quark in any external quark line (a line which is attached at its two ends to external particles). The amplitude remains the same, although the physical process is different. By summing over all types of quarks, we get a factor N for each quark loop. Whenever one of our external particles is a linear combination such as $(1/\sqrt{2})(u\bar{u}+d\bar{d})$ we just take the corresponding linear combination of amplitudes, which is equivalent to using the Chan-Paton factors.[5]

Independently of the SU(N)-symmetry assumption, we must assume that any N^2 reducible multiplet of mesons is degenerate [nonet degeneracy for SU(3)].

In the Appendix we show that this is a fundamental assumption, which is closely related to the assumption that each particle line can be represented as a quark-antiquark pair.

The number N refers only to the observed SU(N) symmetry and not to a color degree of freedom. The presence of color is irrelevant for our discussion since the mesons are color singlets. We cannot sum over color in quark loops, nor can we change the color of an external quark, since in both cases this leads to color nonsinglet mesons.

With these remarks in mind, we now discuss the quark topology structure of the vertex. The vertex as a three-point particle diagram is described in Fig. 3(a). After representing each one of the three particles by a quark-antiquark pair we get Fig. 3(b). Since we are now dealing with only one type of quark ($N=1$), there are two ways to connect the quark lines of Fig. 3(b). The quark of particle a can either go to particle b [Fig. 3(c)] or to particle c [Fig. 3(d)]. Note that we have assumed the Okubo-Zweig-Iizuka[6](OZI) rule for the elementary

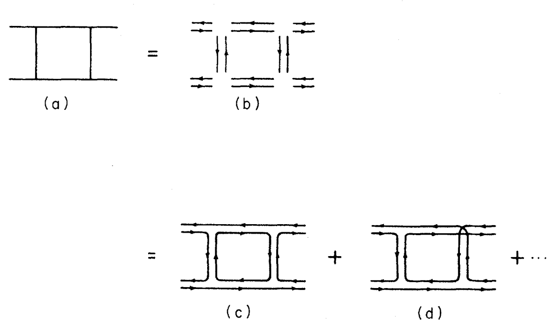

FIG. 2. (a) An example of a particle (or Reggeon) diagram. (b) Diagram (a) assuming quark-antiquark structure for the particles (Reggeons). (c) and (d) Two, out of the 2^4, quark diagrams associated with (a).

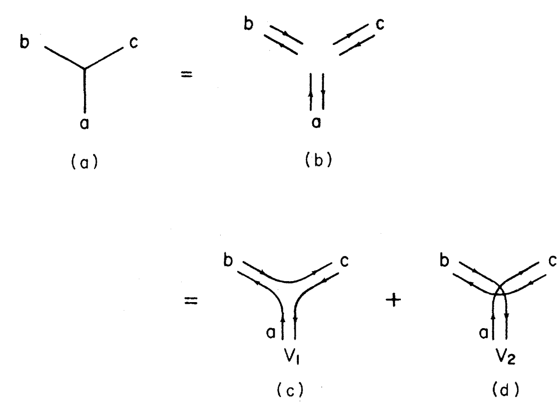

FIG. 3. (a) The vertex as a particle diagram. (b) Diagram (a) assuming quark-antiquark structure for the particles. (c) and (d) The two components of the vertex.

vertex by ignoring contributions of the type of Fig. 4(a). Obviously, this rule is broken when we consider higher-order vertex diagrams, as in Fig. 4(b).

The elementary vertex is therefore composed of two components, which we call V_1 and V_2:

$$V = V_1 + V_2. \tag{1}$$

When we consider any given particle diagram with v vertices [$v=4$ in Fig. 2(a)] we have two components at each vertex, and therefore 2^v terms, each one with a different quark topology. Our task is to find out what is the amplitude associated with each one of these 2^v terms. Therefore, we must know the relation between the two components of each vertex.[7] We derive this relation by applying charge-conjugation (c.c.) transformation to the vertex component V_1. Since we have only one type of quark all three particles are eigenstates of c.c. with eigenvalues C_a, C_b, and C_c. Under the transformation, each particle is multiplied by its c.c. eigenvalue. On the other hand, each quark becomes an antiquark, and therefore in the new vertex component it is the antiquark of a which goes to b. Therefore, the V_1 is transformed into a V_2 component. Because of c.c. conservation, the amplitude is unchanged under this transformation and we get

$$V_2 = C_a C_b C_c V_1 \tag{2}$$

or

$$V_2 = V_1 \text{ (the vertex is allowed by c.c.)}, \quad V_2 = -V_1 \text{ (the vertex is forbidden by c.c.)}. \tag{3}$$

When the process $a \to bc$ is forbidden by c.c., the full vertex $V = V_1 + V_2$ is zero. In fact, this cancellation is the mechanism which is responsible for charge-conjugation conservation. However, each component separately does not vanish (unless it is forbidden by other quantum numbers).

When we compute the full amplitude of a given particle diagram, we sum over all 2^v terms, so we take at each vertex $V_1 + V_2$. Therefore, vertices which are c.c.-forbidden do not contribute. However, when we calculate the contribution of one single term (out of the 2^v terms), we have at each vertex either V_1 or V_2. Therefore, there is no cancellation, and vertices forbidden by c.c. should be included. The conclusion is that whenever we are interested in quark topology, we have to take into account particle diagrams with c.c.-forbidden vertices. Only when we sum over all quark topologies to get the physical amplitude do these diagrams disappear.

The relation between the 2^v terms is now obvious. The only difference between any two terms is at most a minus sign (there is a minus sign for each V_2 component associated with a c.c.-forbidden vertex).

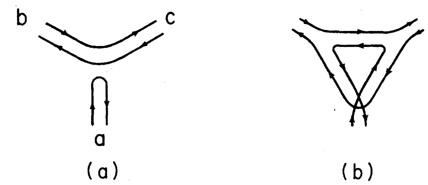

FIG. 4. (a) An example of a vertex component which we neglect (on the basis of the OZI rule). (b) A higher-order vertex diagram which violates the OZI rule.

When we start with a particle diagram which is allowed by c.c., there is no minus sign. Once we know how to calculate the amplitude A of this diagram, we know the contribution of each one of its quark diagrams: $(1/2^v)A$. When the particle diagram is forbidden by c.c., $A=0$, and we cannot calculate each quark diagram. However, we do know that they are all equal in magnitude and we know their relative signs. We need some more information to calculate each term, and as is shown in Sec. III, we do have this extra information in the case of multi-Regge diagrams.

In principle, whenever we are interested in quark topology, we must specify for each choice of particles a, b, and c the V_1 component. Then using Eq. (3), we get the information about V_2 and V. Since the charge-conjugation operator commutes with the spin-space operators, both V_1 and V_2 satisfy angular momentum and parity conservation (unlike c.c. conservation which is satisfied only by their sum).

If two of the particles are identical (say b and c) we get a peculiar behavior when the vertex is c.c.-forbidden, We show in the Appendix that in this case V_1 (or V_2) are antisymmetric in b and c. This does not contradict Bose statistics, since $V = V_1 + V_2 = 0$.

We have just seen that all the 2^v quark diagrams associated with a single particle diagram have the same amplitude (up to a possible minus sign). This seems to contradict our expectation that different quark topologies will have a different analytic structure. At this point we have to notice that until now we considered a single particle diagram, and specified both its external and internal particles. The only hope to get a different analytic structure for different quark topologies is by summing over all possible internal particles. Obviously, for this "miracle" to happen, we must sum over particles with different charge-conjugation properties. (If we take two sets of internal particles, with the same c.c. assignment, we will get the same relative signs of the 2^v quark diagrams so all terms will still be equal in magnitude even after summing over the two sets.)

We shall now demonstrate how such a "miracle" occurs in a very simple example. But first we have to make the following observation: The number of vertices which are forbidden by charge conjugation is even (odd) if the number of external particles with negative charge conjugation is even (odd). This statement is independent of the c.c. assignment of the internal lines. Therefore, if we take any given quark diagram, and make a $V_1 \leftrightarrow V_2$ transformation at each vertex, the new diagram (in which the roles of quark and antiquark lines are interchanged) has the same value if the process is allowed by charge conjugation, and a relative minus sign if the process is forbidden. Therefore, when a process is c.c.-forbidden from the point of view of the external particles, then for any given particle diagram the 2^v corresponding quark diagrams cancel in pairs. For a c.c.-allowed process, any two quark diagrams which are related by a $V_1 \leftrightarrow V_2$ transformation are equal, and it would be sufficient to consider only $\frac{1}{2}\times 2^v$ diagrams.

Consider now the simplest Reggeon diagram of Fig. 5(a). The four external particles are identical and spinless. The exchanged Reggeon is either a tensor (T) with $C=+$, or a vector (V) with $C=-$. Since we have only one type of quark ($N=1$), there is only one T and one V (neglecting lower-lying trajectories).

When we exchange a $C=+$ object (T), all vertices are allowed by charge conjugation. Bose statistics implies that the amplitude is symmetric in particles a and b and therefore in the s and u variables. The exchanged object is necessarily of even spin and positive parity. We write down the amplitude for the tensor exchange in the symbolic form

$$A_T = (-s)^\alpha + (-u)^\alpha , \tag{4}$$

where the first (second) term represents a term with s-channel (u-channel) discontinuity. Since there are two vertices, there are four quark diagrams, in which the upper and lower vertices are of types (V_1V_1), (V_2V_1), (V_2V_2), and (V_1V_2). The last two diagrams are related to the first two by a $V_1 \leftrightarrow V_2$ transformation. Therefore, we consider only the first two diagrams, which are drawn in Figs. 5(b) and 5(c). [They might be more familiar to the reader in the form of Figs. 1(a) and 1(b).] Since the upper vertex is allowed by c.c., the two quark diagrams have the same expression, given by Eq. (4). (We neglect for simplicity the $1/2^v$ normalization.)

We now consider the exchange of the $C=-$ object (V). The two vertices are forbidden by c.c. and therefore we have an extra minus sign in the Bose-statistics relation. The amplitude associated with the quark diagram of Fig. 5(b) is antisymmetric in s and u, and we use for it the expression $(-s)^\alpha - (-u)^\alpha$. Since $V_1 = -V_2$ for a c.c.-forbidden vertex, Fig. 5(c) has an extra minus sign. The situation is summarized in Table I. Before summing over the two exchanges, the two quark diagrams have the same analytic structure. If we are interested in the contribution of a single exchanged Reggeon (T or V), we have to sum over the different quark topologies. Obviously, the V exchange vanishes. However, if we are interested in the contribution of a well-defined quark topology, we have to sum over the possible exchanges. We get $(-s)^\alpha$ for diagram 5(b), and $(-u)^\alpha$ for 5(c). The desired analytic structure has been achieved.

Note that in order to get this analytic structure we have assumed exchange degeneracy (the same trajectory α and the same residue for T and V exchanges). However, our general prescription of decomposing the particle diagram into 2^v components, and of calculating each component, is independent of the exchange-degeneracy assumption. At the end of Sec. III, we discuss the quark topology structure of multi-Regge diagrams in the absence of exchange degeneracy.

II. THE TWIST

The quark topology sturcture of multi-Regge diagrams is usually described using the twist concept.[8] In this section we discuss the relation between the twist description and our two-component

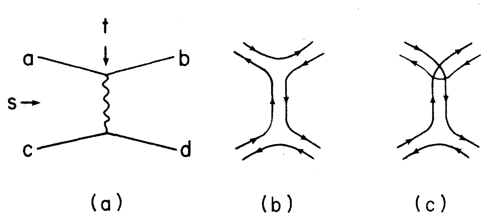

FIG. 5. (a) A Reggeon diagram. (b) and (c) The two quark diagrams associated with (a).

TABLE I. Symbolic amplitudes for tensor and vector exchange in the two quark diagrams associated with the Reggeon diagram of Fig. 5(a).

	Fig. 5(b)	Fig. 5(c)	
T	$(-s)^\alpha+(-u)^\alpha$	$(-s)^\alpha+(-u)^\alpha$	$\Rightarrow (-s)^\alpha+(-u)^\alpha$
V	$(-s)^\alpha-(-u)^\alpha$	$-[(-s)^\alpha-(-u)^\alpha]$	$\Rightarrow 0$
	$\Downarrow$	$\Downarrow$	
	$(-s)^\alpha$	$(-u)^\alpha$	

vertex description. In our method we fix the location of any quark or antiquark line inside an external or internal particle line, as in Fig. 2(b), and then we connect the quark lines in the vertices. The quark lines may cross each other in the vertices, not in the propagators. The situation is reversed in the twist description. Quark lines may cross each other in a propagator (which is then called a twisted propagator) and not in the vertices. The vertex-twist dictionary is very simple. Any V_2 vertex may be deformed, as in Fig. 6(a). There is a twist associated with each external or internal particle attached to a V_2 vertex. Two twists in an internal propagator are equivalent to no twist [Fig. 6(b)]. Twists in external legs are usually ignored (if the ordering of the quark-antiquark lines is not important). Combining these observations, we get the following rule: A propagator has a twist if it connects a V_1 to a V_2 vertex.

Consider now a diagram with v vertices and i internal lines. At first sight it seems that any propagator can be either twisted or untwisted so that we have 2^i different quark topologies. However, it is well known that some of the twist assignments are forbidden due to the restriction that the number of twists in any particle loop should be even. On the other hand, counting diagrams is most natural in the vertex language. As we have already seen, there are 2^v quark topologies. Each vertex assignment (of V_1 or V_2) is allowed, and therefore leads to an allowed twist assignment when we use our dictionary. In fact, the relation between v, i, and the number of particle loops L is

$$v=i+1-L. \tag{5}$$

When we count in terms of twisted or untwisted propagators, we have $2\times 2^i(\frac{1}{2})^L$ quark diagrams instead of the naive result 2^i. There is a factor of $\frac{1}{2}$ for each particle loop due to the above-mentioned restriction. (This limitation of the twist description is unimportant in the case of tree diagrams, where there are no loops.) The factor of 2 is due to another disadvantage of the twist description. For any given quark diagram, a $V_1 \to V_2$ transformation leads to a new quark topology, where the roles of quark and antiquark lines are interchanged. However, according to our dictionary, the twist assignment is unchanged. There is a twofold ambiguity in the twist notation, and in order to resolve it one has to introduce the concept of orientation.

Using the two-component vertex method, it is very easy to derive the relation between quark diagrams and particle diagrams and to understand their charge-conjugation structure. After we sum over all possible exchanges the result might be

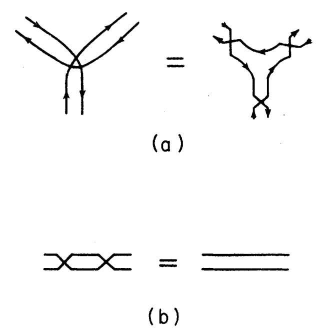

FIG. 6. (a) The relation of the vertex description to the twist description. (b) Two twists = no twist.

simpler in terms of the twist language, as we shall see in Sec. III, since in this language the summation over tensor and vector exchanges is automatically taken into account. However, this simplicity is due to the exchange-degeneracy assumption. Once this degeneracy is lifted, we cannot avoid the summation over V and T exchanges, and it is necessary to use the two-component vertex approach.

It is interesting to note that in some models, it is indeed necessary to deal with Reggeons which are not exchange-degenerate. In Refs. 7 and 9, exchange degeneracy is lifted in the output trajectories of the $2\to 2$ amplitude because of mixing with Pomeron terms. However, once we have Pomeron terms in the output Reggeons, we cannot insert them back in the input $2\to n$ amplitude and use our method, since these break the assumption that all exchanges are of quark-antiquark type.

But if we now take into account the interference terms of the unitarity relation,[4] exchange degeneracy is lifted even before we include the Pomeron terms. The output Reggeon is not a planar object, but is still of a quark-antiquark structure. If we want to insert this non-exchange-degenerate Reggeon as a second approximation input of the unitarity relation, then we have to use the two-component vertex method.

III. MULTI-REGGE QUARK DIAGRAMS

Our starting point in this section is the amplitude for a multiperipheral multi-Regge diagram such as the one in Fig. 7(a). We use the expression for this amplitude, as given in Ref. 10. This expression has been derived using general principles such as analytic structure and Regge behavior, No use has been made of the quark structure of the external particles or of the Reggeons. Our task is to use the method described in Sec. I to derive the amplitude for any given quark diagram associated with the particle diagram of Fig. 7(a). The only extra assumption we need is the quark-antiquark structure of external particles and Reggeons.

The essence of our result is the relation between the signature structure and the quark topology. We consider a diagram with n exchanged Reggeons. The Reggeon i ($i=1,\ldots,n$) can be either a tensor (positive signature, $\tau_i=+$) or a vector (negative signature, $\tau_i=-$). The signature structure of the amplitude for a given set of exchanges $\tau_1\cdots\tau_n$ is

$$A_{\tau_1\cdots\tau_n}=B+\tau_1B^{(1)}+\cdots+\tau_nB^{(n)}+\tau_1\tau_2B^{(1,2)}+\cdots+\tau_1\tau_2\cdots\tau_nB^{(1,2,\ldots,n)}. \tag{6}$$

The term $B^{(1)}$, for example, is related to B by changing the sign of any subenergy variable which overlaps the variable t_1. (t_1 is the "mass" squared of Reggeon 1.) This is the analog of the $s\to -s\sim u$ substitution in the $2\to 2$ amplitude. [The amplitude B is a sum over a few terms. Each term has simultaneous discontinuities in a different set of nonoverlapping variables. However, this analytic structure is irrelevant to our argument. We are only interested in the signature structure given by Eq. (6).]

This signature structure can be derived using the properties of the exchanged Reggeons without assuming anything about their underlying quark structure. (We go to the region where $t_i>0$, and view the diagram as the formation and decay of the Reggeon i. Applying parity transformation to the decay process alone, we get a new amplitude, which is related to the original one by a plus or a minus sign according to the parity of the Reggeon i. This symmetry, when analytically continued to the physical region, leads to the relation between $B^{(1)}$ and B.)

Let us assume that all external particles are of positive charge conjugation. Then at first sight it seems that we do not have the freedom to specify $\tau_1\cdots\tau_n$, since only tensor exchanges ($C=+$) are allowed. The signature structure of Eq. (6) seems to depend on our freedom to choose external particles with both charge conjugation plus and minus. However, this is not the case in our approach. Equation (6) is meaningful even in a world with only one kind of external particles. The diagram of Fig. 7(a) has $n+1$ vertices and therefore 2^{n+1} quark diagrams. As we have already seen in Sec. I, the contribution of any set of exchanges $\tau_1\cdots\tau_n$ to a given quark diagram does not vanish. It is only the sum over all quark diagrams which vanishes, if the given set of exchanges is forbidden by charge conjugation. Therefore, we must interpret $A_{\tau_1\cdots\tau_n}$ [Eq. (6)] as the contribution of this set of exchanges ($\tau_1\cdots\tau_n$) to a well-defined quark diagram. We choose this quark diagram to be the one in which all vertices are of the V_1 type [Fig. 7(b)].

The amplitude for any other quark diagram associated with the same set of exchanges is the same, except for a minus sign for each V_2 vertex which is forbidden by charge conjugation. When we take the only allowed set of exchanges (only tensors), all quark diagrams have the same amplitude, and their sum reproduces Eq. (6) (we omit the $1/2^v$ normalization).

From now on, we shall be interested in a given quark diagram [such as Fig. 7(c)]. In order to calculate its amplitude, we have to find the contribution of any set of exchanges to this specific quark diagram, and then sum over all possible exchanges. Consider a given set $\tau_1\cdots\tau_n$. Its contribution is just $A_{\tau_1\cdots\tau_n}$ with a minus sign for each V_2 vertex forbidden by c.c. Instead, we can associate a minus sign with each edge of a particle (Reggeon) line which is attached to a V_2 vertex, if this particle is of $C=-1$. Therefore, any internal line is associated with a minus sign provided it represents a $C=-1$ Reggeon which connects a V_1 to a V_2 vertex. But according to our dictionary, a propagator which connects a V_1 to a V_2 vertex has a twist. The rule is therefore as follows: There is a minus sign for each $C=-1$ internal particle if its propagator is twisted. In the twist language, the quark diagram we have selected [Fig. 7(c)] is described as in Fig. 7(d). It has twists in Reggeons 1 and 3. We denote the contribution of a given set of exchanges $\tau_1\cdots\tau_n$ to this quark dia-

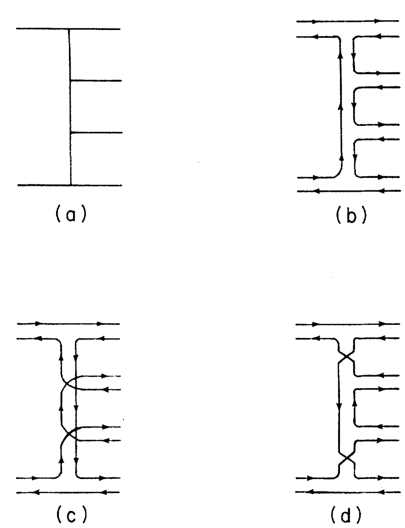

FIG. 7. (a) A multi-Regge diagram. (b) A quark diagram associated with (a), where all vertices are of the V_1 type. (c) A quark diagram associated with (a) with a vertex assignment of $V_1V_2V_2V_1$. (d) Diagram (c) in the twist description.

gram by $A^{(1,3)}_{\tau_1\cdots\tau_n}$. According to our rule

$$A^{(1,3)}_{\tau_1\cdots\tau_n}=\tau_1\tau_3 A_{\tau_1\cdots\tau_n}. \tag{7}$$

From Eq. (6) it is clear that the only term in $A^{(1,3)}_{\tau_1\cdots\tau_n}$ which has no τ coefficients is $B^{(1,3)}$. In order to get the full amplitude for our quark diagram, $A^{(1,3)}$, we have to sum Eq. (7) over all possible sets of exchanges:

$$A^{(1,3)}=\sum_{\tau_1=\pm,\ldots,\tau_n=\pm} A^{(1,3)}_{\tau_1\cdots\tau_n}. \tag{8}$$

Any term with τ coefficients vanishes, so we finally get

$$A^{(1,3)}=B^{(1,3)}. \tag{9}$$

When we have a multiperipheral multi-Regge diagram, with n Reggeons, the amplitude for a given quark diagram with k twists at Reggeons $i_1,\ldots i_k$ is therefore

$$A^{(i_1\cdots i_k)}=B^{(i_1\cdots i_k)}. \tag{10}$$

(A and B depend on n. For simplicity we have omitted the n index and the $1/2^{n+1}$ normalization.)

The first term, B, in Eq. (6) corresponds to the quark diagram of Fig. 7(b). As is shown in Ref. 10, the only discontinuities of this term are in the planar variables of Fig. 7(b). [Any quark diagram associated with Fig. 7(a) defines a cyclic ordering of the external legs. This is the order in which these legs are connected by quark lines. A variable $(p_{j_1}+p_{j_2}+\cdots+p_{j_m})^2$ is called planar if the particles $j_1, j_2, \ldots, j_m$ are next to each other in this ordering.] The discontinuities of any other term, say $B^{(1,3)}$, are in the variables which are planar with respect to the quark diagram (1, 3) of Figs. 7(c) and 7(d). Therefore, our result [Eq. (10)] is that the amplitude of any given quark diagram is just the term in the multi-Regge amplitude [Eq. (6)] which has discontinuities only in the planar variables of this quark diagram. This is obviously the expected result. The two-component vertex method enabled us to establish this expectation. We did not have to use any model (such as the dual model) to derive it, and our only assumption was the quark-antiquark structure of the Reggeons and particles.

Note that this simple result [Eq. (10)] is due to exchange degeneracy. Exchange degeneracy means that the B functions in Eq. (6) do not depend on the set $\tau_1\cdots\tau_n$. The only dependence comes through the τ coefficients (the signature factors). When exchange degeneracy is broken, the modification of our result [Eq (10)] is straightforward. Equation (6) is still valid, but the B functions depend on $\tau_1\cdots\tau_n$ (for example, $B^{(1)}\to B^{(1)}_{\tau_1\cdots\tau_n}$). The relation between $B_{\tau_1\cdots\tau_n}$ and $B^{(1)}_{\tau_1\cdots\tau_n}$ is still as described after Eq. (6). The discontinuities of $B^{(1,3)}_{\tau_1\cdots\tau_n}$, for example, are still in the planar variables of the quark diagram of Figs. 7(c) and 7(d). The contributions of any set of exchanges to this quark diagram is still given by Eq. (7). Therefore, the amplitude of a given quark diagram is given by Eq. (8). We just lose the cancellations which lead to the simple result of Eqs. (9) and (10). Therefore, the amplitude $A^{(1,3)}$ of the quark diagram (1, 3) [Figs. 7(c) and 7(d)] contains terms such as B, $B^{(1,2)}$, etc., which have discontinuities in nonplanar variables. To illustrate this result we consider the case of double-Reggeon exchange ($n=2$), and write down the amplitude for a quark diagram with a twist in Reggeon 1. There are four sets of exchanges: TT, TV, VT, and VV. The contribution of their sum to the specific quark diagram we have selected, $A^{(1)}$, is

$$\begin{aligned}A^{(1)}=\ & B_{TT}+B_{TV}-B_{VT}-B_{VV}\\ &+B^{(1)}_{TT}+B^{(1)}_{TV}+B^{(1)}_{VT}+B^{(1)}_{VV}\\ &+B^{(2)}_{TT}-B^{(2)}_{TV}-B^{(2)}_{VT}+B^{(2)}_{VV}\\ &+B^{(1,2)}_{TT}-B^{(1,2)}_{TV}+B^{(1,2)}_{VT}-B^{(1,2)}_{VV}.\end{aligned} \tag{11}$$

Each row has discontinuities in a different set of variables. Only when we assume exchange degeneracy can we omit the V, T indices and get cancellations for all rows, except for the $B^{(1)}$ row.

The calculation of the B amplitudes is straightforward. We just have to extract from the full amplitude, A, the appropriate τ coefficient. We now derive a recursion formula for the B amplitudes in the physical region.

According to the rules[10] for the multi-Regge amplitude in the physical region, the relation between the n-Reggeon diagram and the $n-1$ is

$$A_{\tau_1\cdots\tau_{n-1}\tau_n}=\frac{\beta(t_n)}{\beta(t_{n-1})}[\Gamma(-\alpha_n)s_n^{\alpha_n}\xi_n]\times(\xi_n^{-1}\xi_{n,n-1}\overline{V}_{n-1}+\xi_{n-1}^{-1}\xi_{n-1,n}\overline{V}_n)A_{\tau_1\cdots\tau_{n-1}}, \tag{12}$$

where

$$\xi_i=e^{-i\pi\alpha_i}+\tau_i$$

and

$$\xi_{i,j}=e^{-i\pi(\alpha_i-\alpha_j)}+\tau_i\tau_j.$$

The vertex function $\overline{V}_n(t_{n-1},t_n,\eta_{n,n-1})$ has a factor of $\eta_{n,n-1}{}^{\alpha_n}$ and $\overline{V}_{n-1}(t_{n-1},t_n,\eta_{n,n-1})$ has a factor of $\eta_{n,n-1}{}^{\alpha_{n-1}}$, where η is the variable related to the Toller angle.

We are now interested in the amplitude of a diagram with a well-defined twist assignment, say $TUUTUT$. T and U stand for twist and untwist. We assume exchange degeneracy and calculate the appropriate B amplitude, which we now denote by B^{TUUTUT} (instead of $B^{(1,4,6)}$). [If exchange degeneracy is broken, we have to take a linear combination as in Eq. (11).] We first have to get rid of the τ factors in the denominators of the right-hand side of Eq. (12), and then extract the appropriate τ coefficient ($\tau_1\tau_4\tau_6$ in the above example). The result is

$$B^{(\cdots)\sigma T}=CB^{(\cdots)\sigma}+DB^{(\cdots)\bar{\sigma}}, \tag{13a}$$

$$B^{(\cdots)\sigma U}=EB^{(\cdots)\sigma}. \tag{13b}$$

This recursion formula is independent of the twist assignment for the first $n-2$ Reggeons which is therefore denoted by $(\cdots)$. We denote the twist assignment of the $n-1$ Reggeon by σ. If σ is T (U), $\bar{\sigma}$ is U (T).

The interesting feature of Eq. (13) is the absence of a $\bar{\sigma}$ term when the last Reggeon is untwisted. In this case the amplitude depends only on the $n-1$ amplitude which has the same twist assignment for the $n-1$ Reggeons. The coefficients in Eq. (13) are given by

$$\begin{aligned} C&=a\frac{\sin\pi(\alpha_{n-1}-\alpha_n)}{\sin\pi\alpha_{n-1}}\bar{V}_n\,,\\ D&=a\left(\bar{V}_{n-1}+\frac{\sin\pi\alpha_n}{\sin\pi\alpha_{n-1}}\bar{V}_n\right),\\ E&=ae^{-i\pi\alpha_n}(e^{i\pi\alpha_{n-1}}\bar{V}_{n-1}+e^{i\pi\alpha_n}\bar{V}_n), \end{aligned} \tag{14}$$

where

$$a=\frac{\beta(t_n)}{\beta(t_{n-1})}s_n^{\alpha_n}\Gamma(-\alpha_n).$$

Because of the presence of the D term in Eq. (13a), there is no twist-untwist factorization. (D cannot vanish identically, since $\bar{V}_{n-1}$ and $\bar{V}_n$ have a different dependence on the Toller variable η.) Anyhow, it is still interesting to note that if we arbitrarily impose a relation between $\bar{V}_{n-1}$ and $\bar{V}_n$, such that $D=0$, then $E:C=e^{-i\pi\alpha_n}:1$. This means that if $D=0$ in some average sense,[4] we get the naive phase prescription: A factor of $e^{-i\pi\alpha}$ is associated with any untwisted propagator, and 1 with any twisted propagator.

SUMMARY

1. Particle diagrams and quark diagrams give a complementary description of hadronic amplitudes. The task of this paper was to compute all quark diagrams associated with any given particle diagram.

2. Each vertex has two components. Therefore, there are 2^v quark diagrams associated with a given particle diagram with v vertices.

3. The two vertex components are related by charge-conjugation symmetry.

4. Vertices forbidden by charge conjugation should be taken into account in order to get the complete information concerning quark topology. There is no contradiction to charge-conjugation symmetry. On the contrary, the mechanism responsible for this symmetry on the particle level is explicitly shown.

5. A single component of a charge-conjugation-forbidden vertex is antisymmetric with respect to two identical particles.

6. All of the 2^v quark diagrams associated with a given particle diagram have the same analytic structure. Only when we sum over all possible exchanges do we get a different analytic structure for each quark topology.

7. A propagator is twisted if it connects two different vertex components.

8. A negative-charge-conjugation exchange has a minus sign if its propagator is twisted.

9. The amplitude of a quark diagram associated with a multi-Regge diagram is given by the term in the full amplitude which has discontinuities in the planar variables of this quark diagram.

10. The above-mentioned result depends on exchange degeneracy. The amplitude of any quark diagram has been also calculated without the exchange-degeneracy assumption.

11. A recursion formula for the amplitude of any quark diagram in the $2\to n$ physical region is given. It exhibits the nonfactorization of the "twist-untwist" amplitude and the factorization of the special quark diagram where all propagators are untwisted. In general, every quark diagram satisfies the same type of factorization provided we go to the physical region associated with its own planar variables.

12. Quark diagrams are useful only if there is N^2 degeneracy for the exchanged objects. This requirement is satisfied provided we exchange objects with a pure quark-anitquark structure, without Pomeron-type corrections.

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APPENDIX

Suppose the SU(N) symmetry is exact. The SU(N) singlet of the $q\bar{q}$ system is not necessarily degenerate with the (N^2-1)-plet. However, quark diagrams are useful only when there is N^2 degeneracy. To demonstrate this, we consider the SU(2) case, where the quarks are denoted by d and

u. The isospin-one neutral meson is $|1\rangle = u\bar{u} - d\bar{d}$ and the isosinglet is $|0\rangle = u\bar{u} + d\bar{d}$. The coupling of a $u\bar{u}$ system to a $d\bar{d}$ system through a meson propagator (Fig. 8) is given by

$$\langle u\bar{u}|(|0\rangle P_0\langle 0| + |1\rangle P_1\langle 1|)|d\bar{d}\rangle, \tag{A1}$$

where P_0 and P_1 are the propagators of the particles with isospin zero and isospin one.

If we want the quark lines to flow through the diagram without changing their identity, we must require this coupling to vanish. This implies $P_0 = P_1$, which means that the isosinglet and the isovector are degenerate.

The great problem is that this desired N^2 degeneracy is inconsistent with quark diagrams themselves. As is well known, diagrams such as Fig. 9(a) contribute to the self-mass correction of the singlet, and decouple from the (N^2-1)-plet.

There is a way out of this problem. In the internal lines of our particle diagrams, we do not insert the full propagators. We use only an approximation to the full propagator, which contains only quark diagrams in which the quark-antiquark lines are going through, as in Fig. 9(b). The planar approximation for the Reggeon[11] is an example of such an approximation.

After throwing away all diagrams of the type of Fig. 9(a), there is no mixing between a $u\bar{u}$ and a $d\bar{d}$ state. Therefore, the most natural set of eigenstates is the $q_i\bar{q}_j$ states $(i,j=1,\ldots,N)$ and not the linear combinations which lead to the SU(N) multiplets. In this stage, SU(N) symmetry does imply the desired N^2 degeneracy, and the problem is solved. The price is using only an approximate propagator. In this approximation the structure of the propagator is of a quark-antiquark pair, and there is no Pomeron-type contamination.

When we have SU(N) symmetry, it is enough to calculate the quark diagrams for the $N=1$ case. Then when we introduce N quarks, we can change the identity of any external quark line and include a factor of N for any closed quark line. It is in this case, of $N=1$, that we have two components at each vertex. But now it is easy to see that the SU(N)-symmetry assumption is not essential. The only essential assumption is that there is no mixing between the $q_i\bar{q}_j$ states in the input propagators. If we break the SU(N) symmetry, we just have to specify the vertex for each set of three particles. Since the particles are of the $q_i\bar{q}_j$ type, there are two kinds of vertices:

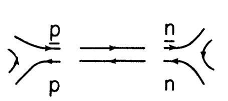

FIG. 8. A coupling of $u\bar{u}$ to $d\bar{d}$.

(1) vertices with only one type of quark $[a(u\bar{u}) \to b(u\bar{u}) + c(u\bar{u})]$, and

(2) vertices with two or three types of quarks $[a(u\bar{u}) \to b(u\bar{d}) + c(d\bar{u})]$, or $[a(u\bar{d}) \to b(u\bar{s}) + c(s\bar{d})]$.

A vertex of the first type has two components (the quark of a can go either to b or to c). Charge-conjugation symmetry relates the two components. In the second type, there is only one possible quark diagram, since the quark of a must go to b. We have only, say, the V_1 component. Applying charge conjugation we get a V_2 component, but of a different process: $\bar{a} \to \bar{b}\bar{c}$. The conclusion is that whenever we have two possible quark topologies in a vertex, they are related by charge-conjugation invariance. Therefore, it is enough to have information about one function per vertex. This conclusion depends strongly on two assumptions: (a) the trilinear meson vertex (ϕ^3) and (b) no violation of the OZI rule.[6] For a ϕ^m meson vertex, there are $m!$ quark topologies. If we impose the analog of the OZI rule (only connected diagrams) we have $(m-1)!$ topologies. In this case the charge-conjugation symmetry relates them in pairs, and we need information about $\frac{1}{2}(m-1)!$ independent functions for every vertex (including vertices which are forbidden by charge conjugation).

Consider now the case where b and c are identical particles. We are interested in the behavior of a single component $V_1^{a(\alpha)\to b(\beta)c(\gamma)}$ as a function of the space-spin degrees of freedom α, β, and γ. When we interchange the roles of b and c, we get the same amplitude, since they are identical particles. However, the quark of a now goes to c. Therefore, we get a V_2 vertex:

$$V_1^{a(\alpha)\to b(\beta)c(\gamma)} = V_2^{a(\alpha)\to b(\gamma)c(\beta)}. \tag{A2}$$

But we have already found [Eq. (3)] that $V_2 = \pm V_1$, so the final result is

$$V_1^{a(\alpha)\to b(\beta)c(\gamma)} = \pm V_1^{a(\alpha)\to b(\gamma)c(\beta)} \tag{A3}$$

(+ when the vertex is allowed by c.c., − when it is forbidden). For a vertex which is forbidden by c.c. ($C_a = -1$), V_1 is *anti*symmetric in the two identical particles. As we have seen, this anomalous behavior is important to get the relation between the charge conjugation and the J^P quantum numbers of particle a. Bose symmetry still holds, since the full vertex, $V = V_1 + V_2$, vanishes.

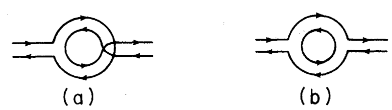

FIG. 9. (a) A self-mass correction which breaks the quark-antiquark structure. (b) A self-mass correction which does not.

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